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Application of the crack-bridging model for fracture resistance (*R*-curve behaviour) to PMMA

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The rising fracture toughness behaviour of PMMA was characterized using a crack-bridging model originally developed for coarse-grained alumina that predicts a mechanism for crack resistance from the bridging of unbroken grains behind the crack tip. Based on the published experimental observation of PMMA, the craze zone behind the crack tip was thought to be analogous to the effective grain-bridging zone in the model in which the fibrils in the craze zone were related to the restraining interfacial ligaments. Self-consistent results in terms of the model were obtained which indicates that the crack-bridging model can be used to account for the fibril-toughening mechanism in PMMA.

1. Introducion

In a previous study [1], we found that PMMA exhibits a rising fracture toughness (*R*-curve behaviour) within an indentation crack size range of 300 to 1300 μm. This behaviour was believed to be caused by crazing near the crack tip. Recently, Mai and Lawn [2] developed a fracture resistance model incorporating crack closure stresses acting behind the crack tip. They applied this model to Al_2O_3 where the closure stresses were caused by unbroken grains acting as "crack bridges" behind the advancing crack front [3]. In an analogous manner, we believe that when a crack in PMMA advances through the craze zone, unbroken fibrils behind the crack tip produce closure forces in a direction opposite to the crack opening stresses (see Fig. 1). From Fig. 1, it can be seen that the effective length of the crack (C) is assumed to include the craze zone and the unbroken fibrils behind the crack tip form a bridging zone of length ΔC. This interpretation of the crack/craze geometry differs somewhat from analyses using the Dugdale model where the crack length is assumed to be up to but not including the craze zone, i.e. C_0 in Fig. 1. Therefore, the purpose of this note is to show the application of the crack closure model [2] in explaining the observed rising fracture toughness of PMMA and discuss the implications that this model has on the crack/craze geometry.

2. Theoretical background

An indentation technique for measuring fracture toughness of PMMA as a function of crack size was described previously [1]. With this technique, a controlled flaw is introduced into a sample with a micro-indentor and the sample is then annealed at 85° C for 8 h to relieve the contact residual stresses. The glass transition temperature (T_g) of this material is approximately 100° C; therefore, it is believed that annealing process did not cause any "healing" or "coalescence" of craze fibrils. By measuring the fracture strength of this sample, fracture toughness can be calculated. The results from the previous study [1] are shown in Fig. 2.

Mai and Lawn's [2] model was developed to explain increasing toughness with crack extension (*R*-curve behaviour) for nontransforming ceramics. They based their model on the experimental observation that grain-localized bridging elements behind the advancing crack tip acted to restrain the crack. The increased crack resistance is a cumulative effect of the crack bridges setting up closure stresses behind the crack tip. They derived an equation for the fracture toughness (T) as a function of effective bridging length (ΔC). The predicted fracture toughness is bounded in the lower limit by some intrinsic toughness (T_0) and in the upper limit by the macroscopic toughness (T_∞). The toughness equations were originally derived for a slit crack, but it can be shown that they are also valid for a penny-shaped crack where:

$$\begin{aligned}
&\Delta C \leqslant d && T = T_0 \\
&d \leqslant \Delta C \leqslant \Delta C^* && T = T_\infty - (T_\infty - T_0) \times \left[1 - \left(\frac{\Delta C - d}{\Delta C^* - d}\right)^{1/2}\right]^{n+1} \\
&\Delta C \geqslant \Delta C^* && T = T_\infty
\end{aligned} \tag{1}$$

where ΔC is the effective bridging length, d is the mean separation between closure force centres, and ΔC^* is the steady state bridging zone dimension. The parameter n is defined as the exponent of the closure stress function [2]. Three n values were examined by Mai and Lawn [2]: $n = 0$ corresponds to a uniform stress distribution along the crack bridging zone; $n = 1$ corresponds to a stress distribution that is a linear function of the crack opening displacement; $n = 2$ corresponds to a non-linear distribution. For

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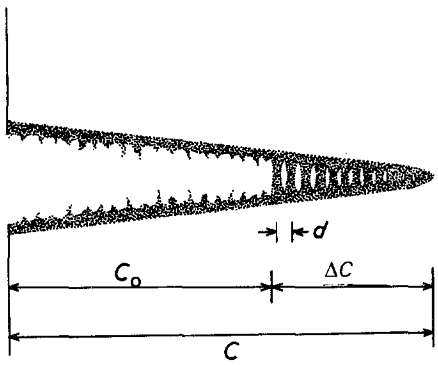

Figure 1 Schematic drawing of crack/craze geometry where C is the effective length of the crack, ΔC is the length of the zone where fibrils bridge the crack faces, C_0 is the length of the crack up to the craze zone.

coarse grained Al_2O_3, Mai and Lawn found the best-fit n value to be 0, i.e. the grain pull-out stresses along the bridging zone are uniform. Note that this model predicts an R-curve behaviour when the effective bridging zone length, ΔC, falls between ΔC^* and d. The toughness will approach either of the two limiting values of T_∞ and T_0 as ΔC approaches ΔC^* and d.

Phenomenologically, the craze zone in relationship to the crack tip in PMMA appears to play a similar role to the crack bridging zone in coarse-grained alumina where the fibrils behind the crack tip are equivalent to the grain-bridging restraint. The critical dimension, ΔC^*, in Mai and Lawn's model [2] would appear to correspond to a steady state craze zone length and the average bridging distance, d, corresponds to the distance between adjacent unbroken fibrils behind the crack tip. With this correspondence, it is of value to investigate the applicability of the crack-bridging toughening model to the predicted fibril-toughening behaviour in PMMA.

3. Results and discussion

Several assumptions based on published PMMA properties have to be made to apply Equation 1 to the experimental data in Fig. 2. Experimental observations of PMMA single edge-notched specimens [4] show that a crack initiates from the initial razor notch and grows stably until catastrophic failure. Such a phenomenon is consistent with Mai and Lawn's model [2] in which the initial crack does not contain "bridges" and starts to propagate at T_0 until final instability is reached at T_∞ where the active bridging zone length becomes equal to the steady state dimension, ΔC^*. The experimental data [4] show that the corresponding T_0 is approximately $0.7\,\mathrm{MPa\,m^{1/2}}$ and T_∞ is about $1.6\,\mathrm{MPa\,m^{1/2}}$. Other experimental data obtained from double cantilever beam specimens [5] give the toughness range 0.6 to $2.2\,\mathrm{MPa\,m^{1/2}}$. This latter range, i.e. $T_0 = 0.6\,\mathrm{MPa\,m^{1/2}}$ and $T_\infty = 2.2\,\mathrm{MPa\,m^{1/2}}$, was taken in the present analysis in order to include all the observed toughness values for PMMA. Experimental results for PMMA also show that crazes grow in length from a crack tip with increasing stress intensity until they reach a limiting value of approximately $38\,\mu m$ [5]. Accordingly, we assumed that the steady state craze zone length, ΔC^*, to be $38\,\mu m$. The distance between two adjacent fibrils d can be obtained using $d = \lambda^{1/2} d_0$ [6], where λ is craze fibrils extension ratio and d_0 is the fibril diameter. For high molecular weight PMMA, the fibril diameter d_0 has been determined to be about $0.025\,\mu m$ [7] and λ to be approximately 2.1 [6]. Therefore, the distance between two adjacent fibrils, d, was calculated to be $0.036\,\mu m$. For the closure stress function along the craze zone, the value of $m = 0$ was chosen, i.e. a uniform stress distribution, which is consistent with the commonly accepted Dugdale model [4, 8, 9].

In our previous study, we found that the indentation

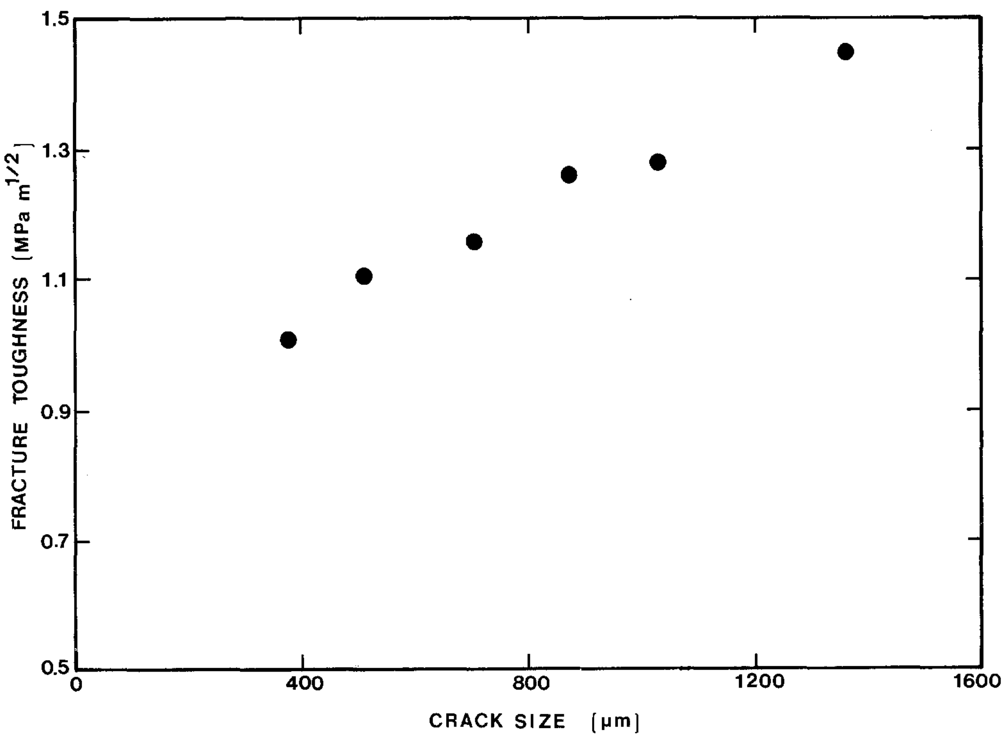

Figure 2 Experimental data of fracture toughness against indentation crack size (from [1]).

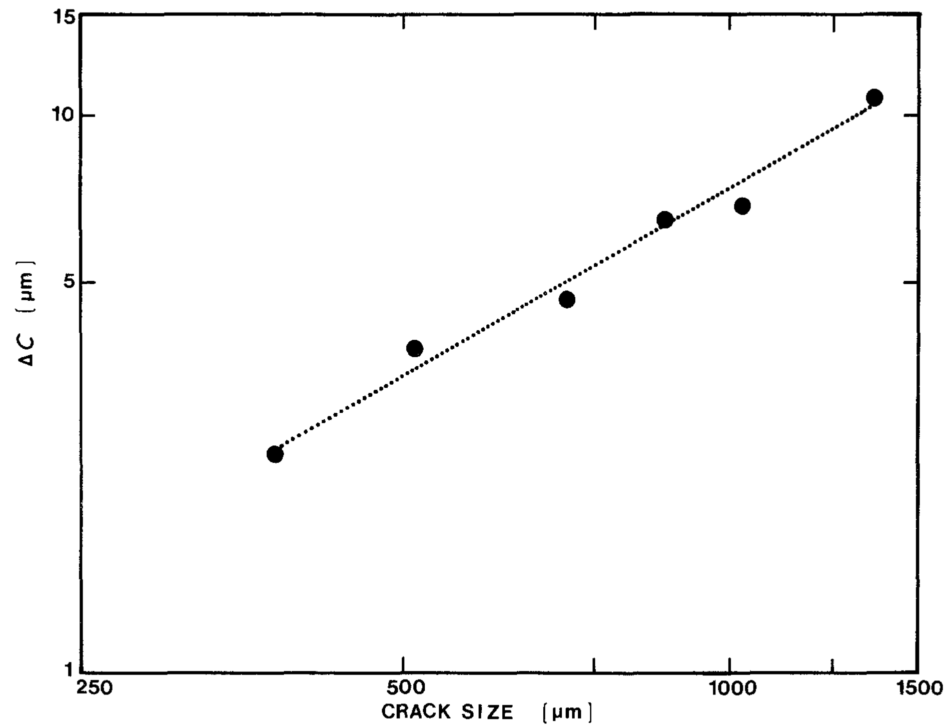

Figure 3 Predicted craze zone length at different crack sizes. (●) Calculated from Equation 1, (···) best fit with slope of 1.143.

crack size, C, increases with indentation load, P. It appears reasonable to assume that the craze zone length, ΔC, also depends on indentation load, P. With this assumption, ΔC will increase as the crack size increases. A relationship between ΔC and C can be obtained by coupling the results in Fig. 2 with the above assumed values for T_0, T_∞, ΔC^*, and d in Equation 1. Fig. 3 shows the calculated results and a best-fit function of ΔC against C. This function can be represented by

$$\Delta C = kC^{\alpha} \qquad (2)$$

where constants k and α were determined from linear regression to be 0.0028 and 1.137, respectively. Note that as the crack size, C, is increased from about 300 to 1300 μm, the fibril bridging zone, ΔC, is predicted to increase from 2 to 10 μm.

We can now use the predicted functional form for ΔC from Equation 2 to obtain a prediction for the R-curve of PMMA by substituting Equation 2 into Equation 1. The results are shown in Fig. 4 along with the original experimental data. The good agreement between the predicted curve and the experimental data follows, of course, from the determination of ΔC against C; however, we can conclude that the crack-bridging model does describe the observed R-curve

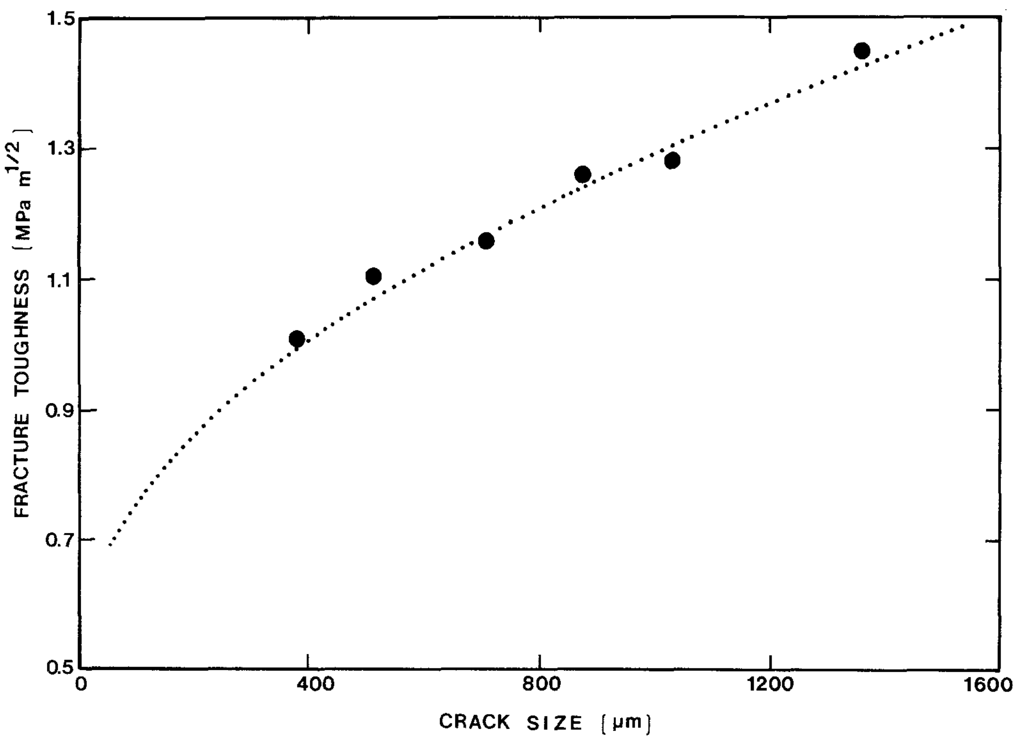

Figure 4 Comparison of the predicted R-curve of PMMA from the crack-bridging model with the fracture toughness data for PMMA from [1].

of PMMA which gives credence to our assumptions regarding fibril-toughening of PMMA.

Mai and Lawn [2] also gave equations for estimating the stress for fibril rupture and the crack opening displacement at the position of fibril rupture. The crack opening at fibril rupture, u^*, a penny-shaped crack can be obtained from [10]

$$\Delta C^* = 2d + (xEu^* / 8^{x/2} \psi T_0)^2 \quad (3)$$

where E is the modulus of elasticity (~ 3.1 GPa), and ψ is the crack geometry constant ($\sim \pi^{1/2}$). The fibril rupture stress, σ^*, can be determined from [2]

$$T_\infty = T_0 + E\sigma^* u^* / T_0 \quad (4)$$

From Equation 3, u^* is calculated to be 2.1 μm and from Equation 4, σ^* is calculated to be 147 MPa. These values compare quite favourably with those found for maximum displacement (1.3 μm) and craze stress (120 MPa) based on an analysis of the crack tip craze zone in PMMA using the Dugdale model [11]. In addition, the product $2u^*\sigma^*$ which represents the work per unit area to separate the fibrils across the fracture plane can be calculated to be $617\,J\,m^{-2}$. This agrees very well with typical fracture surface energies for PMMA (400 to $800\,J\,m^{-2}$) [12]. These results give further confidence in the present analysis and the assumptions made. However, we realize that this does not constitute absolute proof of the model; further research is needed for this, especially an independent measurement of the dependency of the craze-bridging zone with crack length.

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